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EFFECT OF PROBE GEOMETRY ERRORS ON FIELD HARMONIC MEASUREMENTS WITH A ROTATING COIL PROBE

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I INTRODUCTION

The technique being used to make precision measurements of the harmonic content of Energy Saver dipole and quadrupole magnets is that of the rotating coil.[1] The objective is to measure deviations from the main field(either dipole or quadrupole) as small as 1 part in 10^5. In principle, a single planar coil, whose transverse center does not coincide with the center of rotation (rotation along an axis in the beam direction) will suffice. The Fourier components of the induced voltage signal when the coil is rotated through one turn are simply related to the harmonic coefficients of the magnetic field. In practice one has a problem of dynamic range with the "voltmeter" due to the large size of the component from the main field. This can be overcome by having two coils as in Fig. 1 (usually referred to as "inner" and "outer" coils) with outputs summed together such that the main component cancels out; in order to see the radial dependence, the coils must be located at different effective radii. In a physical probe with two coils it is not possible to achieve cancellation to 1 part in 10^5; for the probes in use, the coils are non-coplanar by about 1 degree. To accomplish this a third coil is added at 90^0 to the other two (the "skew" or "side" coil) and the appropriate fraction of its signal is summed in with that of the other two.

Currently, the harmonic coefficients are calculated from the Fourier components assuming a probe of idealized geometry.[2,3] It is the purpose of this note to give an exact formula for the actual design geometry of the dipole harmonic probe and also to derive a more general expression that allows for a limited class of geometrical errors in the probe. The motivation for this study is the experimental fact that the skew coefficient of the 18 pole is non-zero when averaged over many magnets. The normal 18 pole coefficient is known - and expected - to be large ($-13\times10^{-4}\text{in}^{-8}$); the skew is expected to be zero on average.

II PROBE WITH GEOMETRY ERRORS

Fig. 2 shows a "J" type probe geometry with two plausible types of geometrical errors: (a) the center of the inner coil is displaced along a radius perpendicular to the plane of the coil by a distance Δ, (b) the plane of the outer coil does not contain the center of rotation P, but makes an angle α with respect to a radius through the small-radius leg of the coil. *(This leg is assumed to be in the plane of the inner coil.)* Although the errors are completely defined by Δ and α, it is useful to define the other parameters as shown in Fig. 2; some relations among them are:

$$r_0^{\,2} = r_3^{\,2} + W^2 + 2r_3 W \cos\alpha \doteq (r_3 + W)^2 - r_3 W\alpha^2$$

$$\tan\gamma = \frac{W \sin\alpha}{r_3 + W\cos\alpha}; \qquad \frac{\gamma}{\alpha} \doteq \frac{W}{r_0} \tag{1}$$

$$\tan\delta = \frac{\Delta}{w/2}$$

The geometrical constants of the "J" probes [4] are:

$$r_3 = 0.340'' \;,\; r_0 = 0.918'' \;,\; W = w = 0.580'' \;,\; \varepsilon = 0.335''.$$

Our convention here is that positive angle is in the counter-clockwise directions.

III MEASUREMENT PROCEDURE WITH ROTATING COIL

The signals from the three coils (O, I, S stand for outer, inner, and skew) are added together in a resistor network in the "bucking box"; the sum signal, e_Σ goes to a voltage integrator.

$$e_\Sigma = k_0 \left(e_0 + \frac{k_I}{k_0}\, e_I + \frac{k_S}{k_0}\, e_S\right)$$

With the dipole field at 1 T, one adjusts the signal fractions k_0 and k_S such as to minimize the dipole signal at the output of the integrator when the coil is rotated. The integrator output, E, is zeroed at the start of rotation; at some angle Θ, one has

$$E(\Theta) = \frac{k_0}{RC}\left[\Phi_0(\Theta) + \frac{k_I}{k_0}\Phi_I(\Theta) + \frac{k_S}{k_0}\Phi_S(\Theta)\right] - \frac{k_0}{RC}\left[\Phi_0(0) + \frac{k_I}{k_0}\Phi_I(0) + \frac{k_S}{k_0}\Phi_S(0)\right] \tag{2}$$

where Φ is the magnetic flux through a coil. One then substitutes equivalent resistors ($\sim$360Ω) for the inner and side coils and does a rotation with the outer coil alone. ("unbucked" run).

$$E_D(\Theta) = k_0 \frac{B_0 LWm}{RC} \left[\cos(\Theta+\alpha_0) - \cos\alpha_0 \right] \qquad (3)$$

One defines the "standard amplitude ," as

$$A_s \equiv k_0 \frac{B_0 LWm}{RC} \qquad (4)$$

where L is the coil length, m the number of turns in the coil, and RC is the integrator constant. One also chooses the "zero" of the Θ coordinate such that $\alpha_0 = 0$.

To measure harmonics at same dipole field B_0, one then switches I and S coils back in (bucked condition) and makes one coil rotation recording $E(\Theta)$ at 1024 equally spaced points around the circle. $E(\Theta)$ can then be Fourier analyzed in the interval $\Theta=0$ to 2π to give:

$$E(\Theta) = \sum_{N=0}^{14} A_n \cos\left((N+1)\,\Theta + \alpha_n \right) + \text{CONSTANT} \qquad (5)$$

If the bucking is perfect, the dipole term $A_0 = 0$. What remains now is to calculate the right-hand side of eq. (2) in terms of the harmonic expansion of the field and the probe geometry.

IV FLUX CALCULATION THROUGH THE PROBE AND THE RELATION TO HARMONIC COEFFICIENTS

We need to calculate the net flux through the probe when summed over the three coils with the condition that $\Phi_B(\Theta)=0$, if only the dipole field

$$\Phi_B(\Theta) = \Phi_0(\Theta) + \frac{k_I}{k_0}\Phi_I(\Theta) + \frac{k_S}{k_0}\Phi_S(\Theta) \qquad (6)$$

is present. In the standard Energy Saver coordinate system and notation, the harmonic expansion of the two-dimensional field in the body of the magnet is given by

$$B_y + i\,B_x = B_0 \sum_{N=0}^{\infty} C_n Z^n \; ; \quad Z = x + iy \qquad (7)$$

with

$$C_n = b_n + i\,a_n$$

Using this expression for the field one can integrate over the surfaces of the coils in the probe and add them up to obtain an expression for $\Phi_B(\Theta)$ of the form

$$\Phi_B(\Theta) = RC \frac{A_s}{W} \operatorname{Re} \left\{ \sum_{N=0}^{\infty} \frac{C_n}{N+1} r_0^{N+1} e^{i(N+1)\Theta} (R_N + i I_N) \right\} \tag{8}$$

where R_N and I_N are both real functions of r, and depend on the geometry of the probe; the zero of the Θ coordinate is determined by the dipole field as mentioned above. Using this expression in eq. (2) we now have a second equation for $E(\Theta)$ in terms of the b_N's and a_N's. Equating corresponding terms in cos $(N+1)\Theta$ between this equation and eq. (5) yields:

$$b_N = \frac{(N+1)W}{r_0^{N+1}} \frac{A_N}{A_s} \frac{\cos(\alpha_N - \phi_N)}{\sqrt{R_N^2 + I_N^2}} \tag{9}$$

$$a_N = \frac{(N+1)W}{r_0^{N+1}} \frac{A_N}{A_S} \frac{\sin(\alpha_N - \phi_N)}{\sqrt{R_N^2 + I_N^2}}$$

where $\tan\phi_N = I_N/R_N$.

Their ratio becomes:

$$\frac{a_N}{b_N} = \tan(\alpha_N - \phi_N) \tag{10}$$

For a probe with no geometrical errors we will find $I_N = 0$, hence $\phi_N = 0$.

V GENERAL EXPRESSIONS FOR R_N and I_N

Taking the geometry of the probe to be that shown in Fig. 2, integrating over all three coils (all coils have 20 turns), one arrives at the exact expression

$$(R_N + i I_N) e^{i(N+1)\alpha} = e^{i(N+1)\gamma} - \left(\frac{r_3}{r_0} \right)^{N+1} \tag{11}$$

$$- \frac{W\cos(\alpha+\beta)}{w}\left(\frac{w}{2r_0}\right)^{N+1}\left[e^{i(N+1)\delta} + (-1)^N e^{-i(N+1)\delta}\right] e^{-i(N+1)\beta}$$

$$-i\,\frac{W\sin(\alpha+\beta)}{\varepsilon}\left(\frac{\varepsilon}{2r_0}\right)^{N+1}(-1)^{N/2}\left(1+(-1)^N\right) e^{-i(N+1)\beta}$$

The three lines in eq. (11) are the contributions of the outer, inner, and side coils respectively; the factors $\frac{W}{w}\cos(\alpha+\beta)$ and $\frac{W}{\varepsilon}\sin(\alpha+\beta)$ arise from the cancellation condition for the dipole field.

In the region where $\alpha, \delta << \frac{1}{N+1}$ eq. (11) gives (letting $W=w$).

$$R_N = 1 - \left(\frac{r_3}{r_0}\right)^{N+1} - \left(1+(-1)^N\right)\left(\frac{w}{2r_0}\right)^{N+1} \tag{12}$$

$$I_N = (N+1)\left[-(\alpha-\gamma) + \alpha\left(\frac{r_3}{r_0}\right)^{N+1} - \left(1+(-1)^{N+1}\right)\left(\frac{w}{2r_0}\right)^{N+1}\delta\right.$$

$$\left. + \left(1+(-1)^N\right)(\alpha+\beta)\left(\left(\frac{w}{2r_0}\right)^{N+1} - \frac{W}{\varepsilon}\frac{(-1)^{N/2}}{(N+1)}\left(\frac{\varepsilon}{2r_0}\right)^{N+1}\right)\right]$$

With no restrictions on α and δ (but neglecting the side coil) eq. (11) yields:

$$R_N = \cos\left((N+1)(\alpha-\gamma)\right) - \left(\frac{r_3}{r_0}\right)^{N+1}\cos\left((N+1)\alpha\right)$$

$$- \left(\frac{w}{2r_0}\right)^{N+1}\cos(\alpha+\beta)\left[\left(1+(-1)^N\right)\cos\left((N+1)\delta\right)\cos\left((N+1)(\alpha+\beta)\right)\right.$$

$$\left. + \left(1+(-1)^{N+1}\right)\sin\left((N+1)\delta\right)\sin\left((N+1)(\alpha+\beta)\right)\right]$$

$$I_N = -\sin\left((N+1)(\alpha-\gamma)\right) + \left(\frac{r_3}{r_0}\right)^{N+1}\sin\left((N+1)\alpha\right) \tag{13}$$

$$+ \left(\frac{w}{2r_0}\right)^{N+1} \cos(\alpha+\beta) \Big[(1+(-1)^N) \cos((N+1)\delta) \sin((N+1)(\alpha+\beta))$$

$$- (1+(-1)^{N+1}) \sin((N+1)\delta) \cos((N+1)(\alpha+\beta)) \Big]$$

Hence for large N

$$\tan \phi_N = I_N / R_N = - \tan((N+1)(\alpha - \gamma))$$

or

$$\phi_N = -(N+1)(\alpha-\gamma)$$

Numerical values for the "J" probes for the quantities appearing in the formulae for R_N and I_N are:

$$\frac{r_3}{r_0} = \frac{1}{2.70} \quad ; \quad \frac{w}{2r_0} = \frac{1}{3.17} \quad ; \quad \frac{w}{\varepsilon} = 1.73 \quad ; \quad \frac{\varepsilon}{2r_0} = \frac{1}{5.48}$$

With this in mind, some observations that can be made from eq. (12) are:

1. For a "perfect" probe, $I_N = 0$

2. For $N \geq 4$ (decapole), it is a good approximation (1% level) to keep only the first term in both R_N and I_N

   $$R_N = 1 \quad , \quad I_N = - (N+1)(\alpha - \gamma)$$

   Therefore, the only parameters of the probe that enter are w, r_0, α, and γ; i.e. the inner and side coils do not enter. Angle α enters explicitly because it determines the phase of the dipole term from the outer coil.

3. For $N \geq 4$ the dominant effect of the imperfect probe, is an apparent rotation of higher multipoles relative to the dipole by an angle $-(N+1)(\alpha-\gamma)$; for small α_N the biggest effect is seen in a_N (see eq. (9).

4. For $N \leq 4$, the parameters of the inner and side coils begin to play a role; even for the smallest N (N = 2) to which the side coil contributes, it is negligible.

5. For $\Delta \neq 0$ (hence $\delta \neq 0$), one can change the sign of $(\alpha - \gamma)$ without changing the sign of $(\alpha + \beta)$, the angle between the planes of inner and outer coils.

VI COMPARISON OF FORMULAE USED FOR CALCULATING a_N AND b_N

A. Probe without geometrical errors.

Until May, 1982, the formulae in use at MTF for the "J" probes were:

$$b'_N = \frac{2}{3} \frac{(N+1) \quad A_N}{r_0^N \quad A_s \quad f_N} \cos \alpha_N \tag{14}$$

$$a'_N = b_N \tan \alpha_N$$

where

$$f_N = 1 - 2 \left(\frac{1}{3}\right)^{N+1} + \left(\frac{-1}{3}\right)^{N+1}$$

From eq. (9) and eq. (12) we find:

$$b_N = \frac{(N+1)\ W}{r_0^{N+1}} \frac{A_N}{A_s} \frac{1}{R_N} \cos \alpha_N \tag{15}$$

$$a_N = b_N \tan \alpha_N$$

where

$$R_N = 1 - \left(\frac{r_3}{r_0}\right)^{N+1} - (1 + (-1)^N) \left(\frac{w}{2r_0}\right)^{N+1}$$

Egs. (14) follow from eqs. (15) in the approximation (W = w)

$$\frac{W}{r_0} = \frac{2}{3} \quad , \quad \frac{r_3}{r_0} = \frac{1}{3}$$

where in actuality $\frac{W}{r_0} = 0.632$, $\frac{r_3}{r_0} = \frac{1}{2.7}$.

Forming the ratio gives

$$\frac{b_N}{b'_N} = \frac{3}{2} \frac{W}{r_0} \frac{f_N}{R_N} = 0.948 \frac{f_N}{R_N}$$

Table I exhibits b_N / b'_N vs. N for the J probe and also for the obsolete A-style probe [2,3,4]:

TABLE I

N	b_N / b_N' J PROBE	b_N / b_N' A PROBE
1 (QUAD)	0.976	0.961
2	0.951	0.942
3	0.954	0.933
4	0.949	0.927
5	0.949	0.924
6	0.948	0.922
7	0.948	0.920
8	0.948	0.919
9	0.948	0.919
10	0.948	0.918

Hence for the "perfect" J probe, b_N' and a_N' are too large by $\sim$5% and for the older A probe $\sim$8%.

B. Probe With Geometric Errors

In May, 1982, a probe "flattening" correction was added to eq. (14) with

$$\cos \alpha_N \rightarrow \cos (\alpha_N + N (\alpha + \beta))$$

$$\tan \alpha_N \rightarrow \tan (\alpha_N + N (\alpha + \beta)) \qquad (16)$$

where $(\alpha+\beta)$ (see Fig. 2) is the angle between the planes of the inner and outer coils, which can be measured directly and is $\sim 1^\circ$. We find from eqs. (9) and (13) that for $N \geq 4$ the corrections should be

$$\cos\left(\alpha_N + (N+1)(\alpha - \gamma)\right) \tag{17}$$

$$\tan\left(\alpha_N + (N+1)(\alpha - \gamma)\right)$$

i.e., the angle β does not enter in the phase correction. As noted in eq. (1), α and γ are related by geometry such that $(\alpha - \gamma) = 0.37\alpha$. A recent study by A. Wehmann [5] on the phase of the 18 pole amplitude indicates that $(\alpha - \gamma) = 1.2^0$ for the probe labelled "J2."

VII SUMMARY OF FORMULAE FOR a_N AND b_N FOR J-TYPE PROBE

Gathering together eqs. (9), (10), and (12) we have (geometric parameters are defined in Section II above)

$$b_N = \frac{(N+1)\,W}{r_0^{N+1}\,R_n}\;\frac{A_N}{A_s}\;\frac{\cos\left(\alpha_N - \phi_N\right)}{\sqrt{1 + I_N^2/R_N^2}}$$

$$a_N = b_N \tan\left(\alpha_N - \phi_N\right)$$

$$\tan\phi_N = I_N / R_N$$

For $\alpha,\ \delta \ll \frac{1}{N+1}$; (18)

$$R_N = 1 - \left(\frac{r_3}{r_0}\right)^{N+1} - \left(1 + (-1)^N\right)\left(\frac{w}{2r_0}\right)^{N+1}$$

$$I_N = (N+1)\left[-(\alpha - \gamma) + \alpha\left(\frac{r_3}{r_0}\right)^{N+1} - \left(1 + (-1)^{N+1}\right)\left(\frac{w}{2r_0}\right)^{N+1}\delta \right.$$
$$\left. + (\alpha + B)\left(1 + (-1)^N\right)\left\{\left(\frac{w}{2r_0}\right)^{N+1} - \frac{W}{\varepsilon}\,\frac{(-1)^{N/2}}{N+1}\left(\frac{\varepsilon}{2r_0}\right)^{N+1}\right\}\right]$$

Using the known geometry constants of the J probes, we give same numerical results for R_N and I_N:

$R_1 = 0.863, \quad R_2 = 0.886, \quad R_3 = 0.981, \quad R_4 = 0.987, \quad R_5 = 0.997$

$$I_1 = 2 \left[-(\alpha - \gamma) + \frac{\alpha}{7.3} - \frac{\delta}{5.0} \right]$$

$$I_2 = 3 \left[-(\alpha - \gamma) + \frac{\alpha}{19.7} + (\alpha + \beta)\left(\frac{1}{16.0} + \frac{1}{143} \right) \right]$$

$$I_3 = 4 \left[-(\alpha - \gamma) + \frac{\alpha}{53.1} - \frac{\delta}{50.5} \right]$$

$$I_4 = 5 \left[-(\alpha - \gamma) + \frac{\alpha}{143.5} + (\alpha + B)\left(\frac{1}{160} - \frac{1}{7130} \right) \right]$$

Remembering that $(\alpha - \gamma) = 0.368\alpha$, we see that the approximation

$$I_N \approx -\sin\left((N+1)(\alpha - \gamma) \right)$$

should be fairly good for $N \geq 4$ (decapole).

VIII <u>EFFECT OF $\alpha \neq 0$ ON DETERMINING THE COORDINATE FRAME WHERE THE 16-POLE IS ZERO</u>

The harmonic coefficients given by eq. (18) are in an (x,y) coordinate system whose origin is at the center of rotation of the coil. For the dipoles the coefficients are transformed to the system where the 16-pole is zero, ($a_7 = b_7 = 0$). Using this prescription, one finds from the data that the center of rotation of the coil usually lies within the regions:

$$x = 0 \pm 50 \text{ mils}$$

$$y = -65 \pm 50 \text{ mils}$$

Clearly if the probe has the type of geometric error $\alpha \neq 0$, then this prodedure is in error if eq. (14) is used in calculating a_N and b_N.

One can obtain some idea of possible errors involved by the following procedure: (1) assume that the dipole has only one non-zero higher harmonic, $b_8 = -13\text{x}10^{-4}\ (\text{in})^{-8}$ in the central reference frame, (2) transform this into a frame displaced by (x, y), (b_7 and a_7 will be generated), (3) rotate the multipoles c_7 and c_8 by an angle $(N+1)(\alpha-\gamma)$, and (4) calculate a translation which will make $a_7 = b_7 = 0$. In general one does not return to the original coordinate frame; in first order the errors are

$$\Delta y = x \sin (\alpha - \gamma); \Delta x = y \sin (\alpha - \gamma) \tag{19}$$

Hence if the probe has a y displacement of 120 mils and $(\alpha - \gamma) = 2^o$, then the $\sim$ shift will be in error by 4.2 mils. This does not appear to be a big effect (e.g., it will feed 0.8% of the sextapole into the quadrupole); on other hand, a real dipole has many more non-zero harmonics and it may be that the error is sometimes significant.

IX CONCLUSIONS

We have derived expressions for calculating the harmonic coefficients of the magnetic field applicable to a rotating coil probe of the "J" type geometry. The "perfect" probe can be described by three geometric constants (only inner and outer coils needed); we have considered a more general case that allows for a limited choice of geometric errors describable by two additional angle parameters (δ and α in Fig. 2). The biggest effect arrises from $\alpha \neq 0$; this occurs when the plane of the outer coil does not contain the center of rotation. If ignored, this error rotates the higher multipole amplitudes, c_N, relative to the dipole by an angle $\sim 0.4\ \alpha$ (N+1). Such a rotation will generate skew coefficients, a_n, if the normal coefficient, b_N, is non-zero. For the probe labelled J2, the value of a_8 averaged over many magnets indicated a value of α of $\sim 3^o$, assuming it is due to this type of geometric error.

Other conclusions arrived at during the course of this study are:

(a) The formulae in use at MTF for calculating b_N and a_N assume an idealized probe geometry which differs appreciably from the design geometry. The effect of using this approximation is that calculated coefficients are 5-8% too large (see Table I).

(b) In May, 1982, Dan Gross introduced a phase correction into the calculation of b_N and a_N (called "probe flattening") in an attempt to make $< a_8 > = 0$; he applied a phase correction of $N(\alpha+\beta)$, where $(\alpha+\beta)$ is the angle between the planes of inner and outer coils. As shown above the proper correction is $(N+1)\ (\alpha - \gamma)$ (which equals $0.37(N+1)\alpha$ for our geometry). In our model it is even possible for $(\alpha - \gamma)$ and $(\alpha + \beta)$ to have opposite signs. For the higher multipoles $(N \geq 4)$, because of the strong r dependence, the inner coil is not relevant.

(c) The harmonic coefficients for a dipole are given in a coordinate frame that gives zero sixteen pole. In practice it is necessary to shift the coordinate system by as much as 0.15 in. to accomplish this. If $\alpha \neq 0$, this shifting will be in error; although the effect appears not to be large, a definitive study should be done with actual magnet data.

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REFERENCES

1. M. Wake, D. A. Gross, and R. Yamada, Cryogenics, 341, (1981).

2. M. Wake, Fermilab TM-958 (1980).

3. M. Wake, Memo to J. Pachnik, "Modification of DCH Program for New Probe," May 12, 1982.

4. D. A. Gross, "Harmonic Probe Calibrations at MTF", June 25, 1982.

5. A. Wehmann, "Corrections to Dipole Harmonics - A Progress Report", Aug. 5 (1982); "A Discussion of Phase of the 18 Pole Harmonic Coefficient in the Doubler Dipole", Aug 11 (1982); "Comments Upon the Effect in the 18 Pole Phase", Sept 3 (1982).

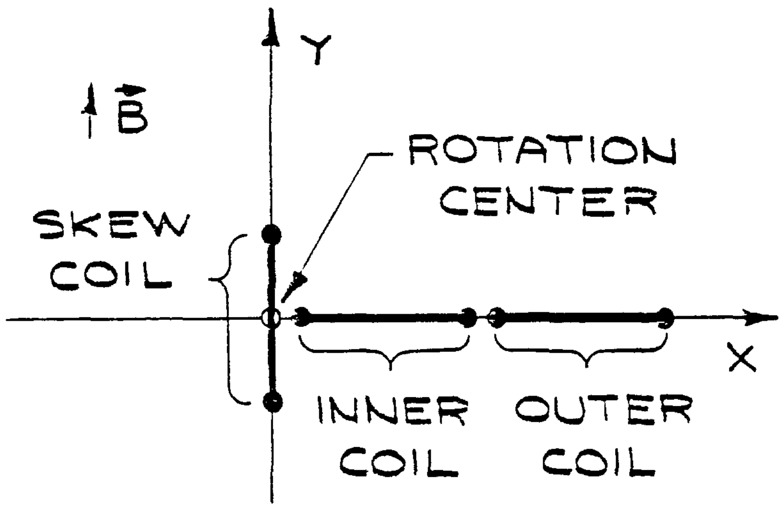

FIGURE 1.

ROTATING COIL HARMONIC PROBE, WITH INNER, OUTER AND SKEW COILS

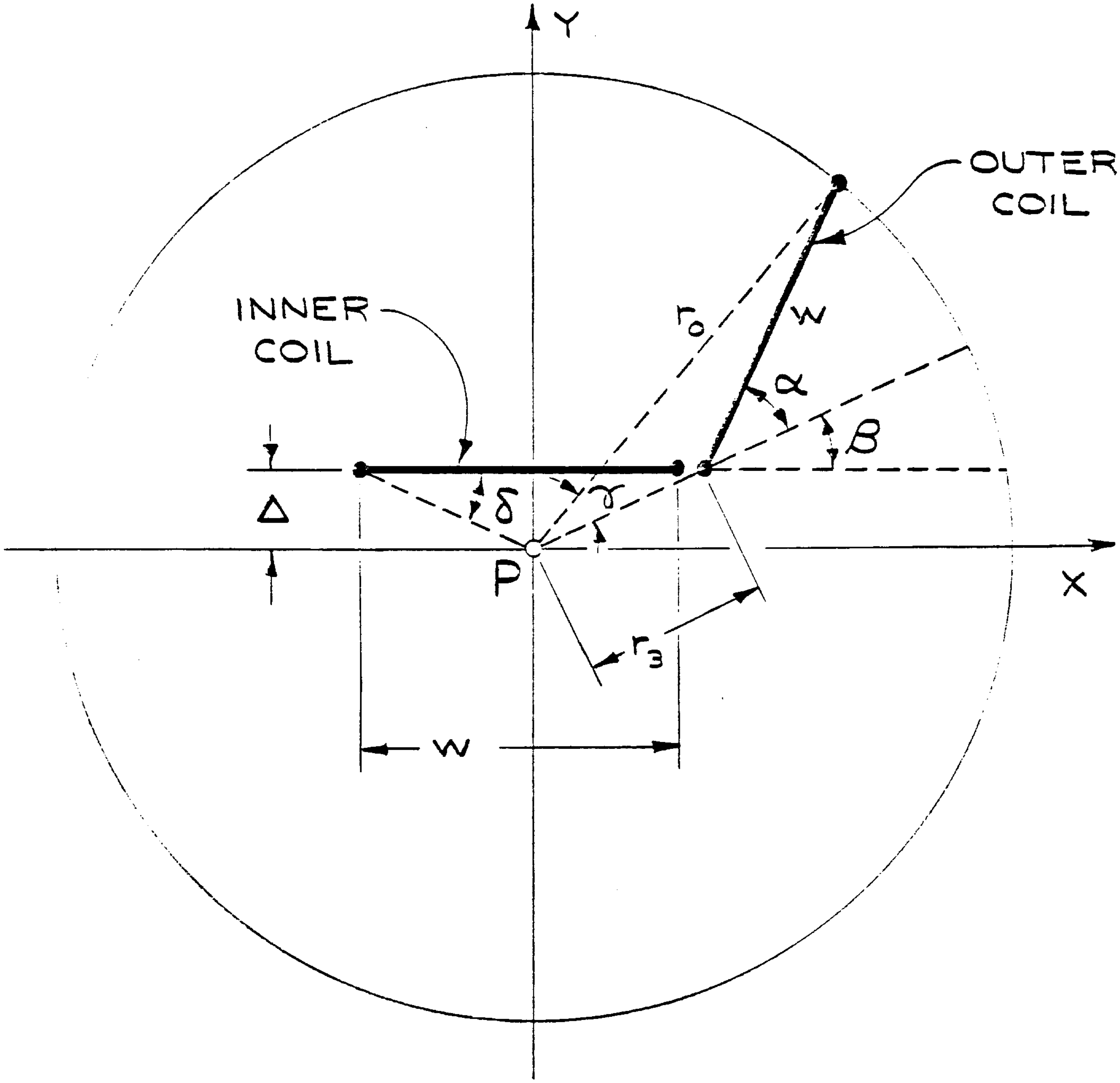

FIGURE 2.

J-STYLE ROTATING COIL PROBE
WITH GEOMETRIC ERRORS Δ AND α